Introduction to Signal and Systems

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1. Express each of the following complex numbers in Cartesian form (x+jy)

1. $\frac{1}{2}e^{j\pi}$

$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$

$e^{i\pi} = \cos pi + j\sin\pi$ = -1 + 0j = - 1

$\frac{1}{2}e^{j\pi}$ = -1/2

2. -$\frac{1}{2}e^{j\pi}$

$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$

$e^{i\pi} = \cos pi + j\sin\pi$ = -1 + 0j = - 1

-$\frac{1}{2}e^{j\pi}$ = 1/2

3. $e^{j\frac{\pi}{2}}$

$e^{\pm i\theta} = \cos\theta \pm j\sin\theta$

$e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2}$ = 0 + j1 = j

$e^{j\frac{\pi}{2}}$ = j

2. Express each of the following complex numbers in polar form $re^{j\pi}$, with -pi $< \theta <$ pi

x+yj

r = $(x^2 + y^2)^{\frac{1}{2}}$

$\theta = tan^{-1}(\frac{y}{x})$

1. 5+0j

r = $(5^2 + 0^2)^{\frac{1}{2}}$ =5

$\theta = tan^{-1}(\frac{0}{5})$ = 0

$5e^{j0}$

2 -2+0j

r = $(5^2 + 0^2)^{\frac{1}{2}}$ = 2

$\theta = tan^{-1}(\frac{0}{-2})$ = π

$2e^{j\pi}$

3. 0+-3j

r = $(0^2 + (-3)^2)^{\frac{1}{2}}$ = 3

$\theta = tan^{-1}(\frac{-3}{0})$ = -$\pi/2$

$3e^{j-\pi/2}$

3. Determine the values of P_∞ and E_∞ for each of the following signals:

E_∞ = $\int_{-\infty}^{\infty} |x(t)|^2 dt$

P_∞ = $\lim_{t\to+\infty} \frac{1}{2T} \int_{-\infty}^{\infty} |x(t)|^2 dt$

x(t) = $e^{-2t}u(t)$

E_∞ = $\int_{-\infty}^{\infty} |e^{-2t}u(t)|^2 dt$

E_∞ = $\int_{0}^{\infty} |e^{-2t}|^2 dt$

E_∞ = $\int_{0}^{\infty} |e^{-4t}| dt$

E_∞ = $\frac{e^{-4t}}{-4}\Big|_0^\infty$

E_∞ = $\frac{1}{4}$

P_∞ = $\lim_{t\to+\infty} \frac{1}{2T} E_\infty$

P_∞ = $\frac{1}{\infty}\frac{1}{4}$ = 0

4. Let x[n] be a signal with x[n] = 0 for n $<$ -2 and n $>$ 4. For each signal given below, determine the values of n for which it is guaranteed to be zero.

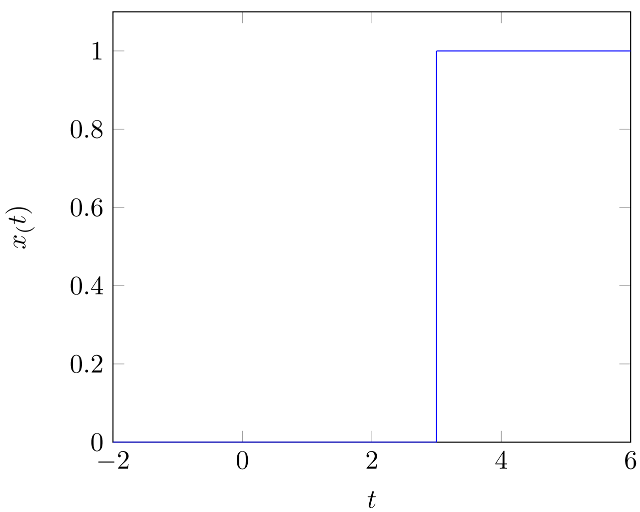
x[n] = 0 for n $<$ -2 and n $>$ 4

a) X[n-3]

This graph shift by three thus
X[n-3] is guaranteed to be zero for $n < 1$ and $n > 7$

5. Let x(t) be a signal with x(t) = 0 for t $<$ 3. For each signal given below, determine the values of t for which it is guaranteed to be zero.

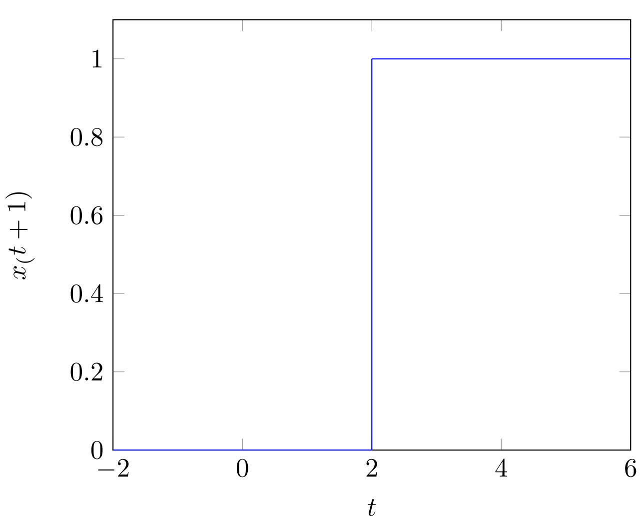
X(t)



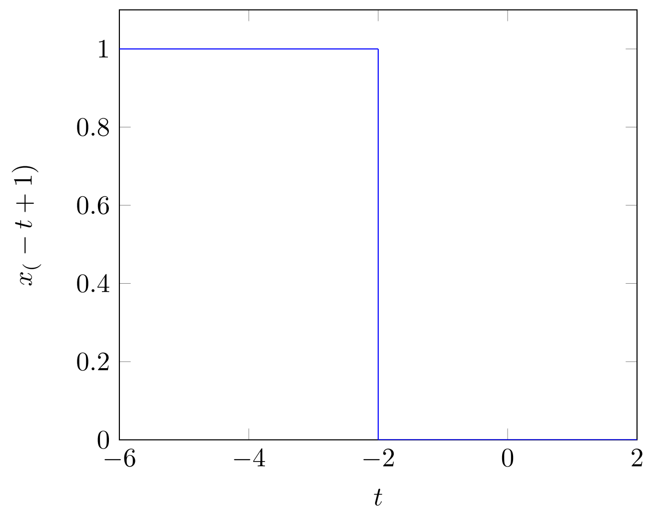
x(1-t) + x(2-t)

x(1-t) = x(-t +1)

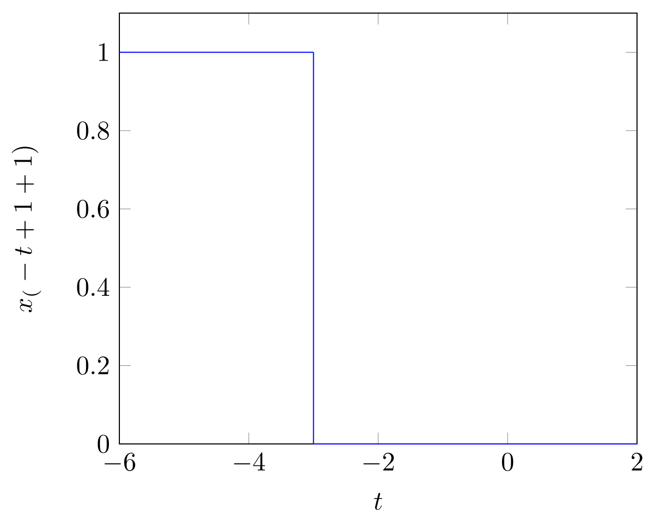
+1 is leftshift by one thus



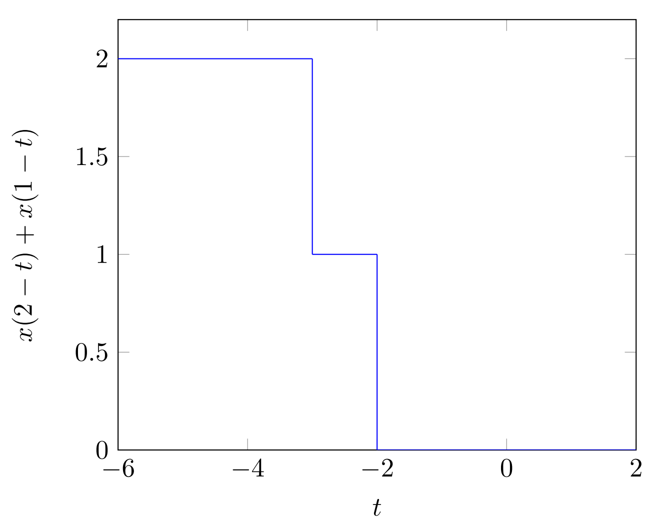
Time reverse of signal x(t+1) gives the signal x(1-t), thus



x(2-t) = x(-t +1 + 1)
thus, it shifts left one from signal x(-t+1)



x(2-t)+ x(1-t) is addition of the two graph



Therefore, the signal x(1-t) + x(2-t) is 0 when t >-1

6. Determine whether or not each of the following signals is periodic

$x_1(t)$ = $2e^{j(t+\frac{\pi}{4})}u(t)$

u(t) is unit step signal and it is zero at t<0, therefore, $x_1(t)$ is defined only for t>0. thus $x_1(t)$ is not periodic.

Q.7: 1.9(a) Determine whether or not each of the following signals is periodic. If a signal is periodic, specify its fundamental period.

$x_1(t)$ = j$e^{j*10(t)}$

Consider j

r = $(0^2+(1)^2)^{\frac{1}{2}}$ = 1

$\theta = tan^{-1}(\frac{1}{0})$ = $\pi/2$

$j = 1e^{j\pi/2}$

$x_1(t)$ = $e^{j\frac{\pi}{2}} * e^{j*10(t)}$

$x_1(t)$ = $e^{j(10(t)+\frac{\pi}{2})}$

ω = 10 $\quad \phi$ = $\frac{\pi}{2}$

T = $\frac{2\pi}{\omega}$

T = $\frac{2\pi}{10}$

T = $\frac{\pi}{5}$

8. Consider the continuous-time signal

x(t) = δ(t + 2) - δ(t - 2)

Calculate the value of E_∞ for the signal

y(t) = $\int_{-\infty}^{t} x(T)dT$

y(t) = $\int_{-\infty}^{t} \delta(t+2) - \delta(t-2)dT$

y(t) = $\int_{-\infty}^{t} \delta(t+2)dT - \int_{-\infty}^{t} \delta(t-2)dT$

y(t) = u(t+2) - u(t-2)

y(t) = 0, for t <-2
y(t) = 1, for -2 <= t <= 2
y(t) = 0, for t>2

E_∞ = $\int_{-\infty}^{\infty} y(t)^2 dt$

E_∞ = $\int_{-\infty}^{\infty} u(t+2) - u(t-2)dt$

E_∞ = $\int_{-\infty}^{-2} 0\,dt + \int_{-2}^{2} 1\,dt - \int_{2}^{\infty} 0\,dt$

E_∞ = 4

9.Consider a discrete-time system with input x[n] and output y[n]. The input-output relationship for this system is

y[n] = x[n]x[n-2]

(A) is the system memoryless?

A system is said to be memory less if its output for each value of the independent variable at a given time is dependent only on the input at that same time.

Example)
A resistor is a memoryless system; with the input x(t) taken as the current and with the voltage taken as the output y(t), An example of a discrete-time system with memory is an accumulator or summer, such as capacitor

Systems whose output y(t0) at time t0 depends on values of the input other than just x(t0) have memory.

since input has other inputs other than x[n], which is x[n-2] , the system has memory. Thus, it is not memoryless

B) Determine the output of the system when the input is Aδ[n], where A is any real or complex number.

y[n] = Aδ[n]*Aδ[n-2]

Aδ[n] is defined at n = 0, and everywhere else is 0

Aδ[n-2] is defined at n =2, and everywhere else is 0

The product of these two Aδ[n] and Aδ[n-2] is zero.because they do not have a common part.

Therefore, the output of the system y[n] is zero.

C) is the system invertible?

A system is said to be invertible if distinct inputs lead to distinct outputs. A system is invertible if you can determine the input uniquely from the output, i.e. there is a one-to-one relationship between the input and output

Definition of one to one relationship
$\forall a, b \in X, \;\; f(a) = f(b) \Rightarrow a = b$

in function y[n] = Aδ[n]*Aδ[n-2], the output is always zero whatever the input is. This is not the one to one relationship. Thus, this function is not invertible

10. Consider a continuous-time system with input x(t) and output y(t) related by

y(t) = x(sin(t))

(a) Is this system causal?

System is casual when Output y(t) depends only on past and present inputs and not on the future.

Definition of casual system

$x_1(t) = x_2(t), \quad \forall\ t < t_0,$

the corresponding outputs satisfy

$y_1(t) = y_2(t), \quad \forall\ t < t_0.$

y(t) = x(sin(t))

when t = 0
y(0) = x(0)

when t = π/2

y(π/2) = x(1)

its shown that the output depends on future value of input, thus it is not casual.

(b) Is this system linear?

y(t) = x(sin(t))

Additivity: f(x + y) = f(x) + f(y).
Homogeneity: f(bx) = b f(x) for all b.

$a_1 * y_1(t)$ + $a_2 * y_2(t)$ = $a_1 * [x_1(sin(t))] + a_2 * [x(sin(t))]$

Equation 1: $a_1 * y_2(t)$ + $a_2 * y_2(t)$ = $a_1 * x_1(sin(t)) + a_2 * x_2(sin(t))$

if x_3= a_1*x(sin(t)) + a_2*x(sin(t))

Equation 2: $a_3 * y_3(t)$ = a_1*x(sin(t)) + a_2*x(sin(t))

Using equation 1 and 2,

$a_1 * y_2(t)$ + $a_2 * y_2(t)$ = $a_3 * y_3(t)$

Thus, system is linear is linear