THE LEPLACE TRANSFORM

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Q.1:(9.1(a)) For each of the following integrals, specify the values of the real parameter σ which ensure that the integral converges

(a)
$$\int_0^\infty e^{-5t}e^{-(\sigma+j\omega)t}dt$$

e^{-at} converges in the interval t: (0,∞), if a>0
e^{-at} converges in the interval t: (-∞,0), if a<0

$$\int_0^\infty e^{-5t}e^{-(\sigma+j\omega)t}dt = \int_0^\infty e^{-5t-\sigma}e^{-j\omega t}dt$$

The maxium value of $e^{-j\omega t}$ is 1, so the convergence of the integral depends on $e^{-(5+\sigma)t}$ the range of t is (0,∞) and for the integral to converge 5+σ $>$ 0 and σ $>$ -5.

Thus, the range of the value of the real parameter σ which ensure that the integral converges is σ $>$ -5.

Q.2:(9.2(a)) Consider the signal

$$x(t) = e^{-5t}u(t-1)$$

and denote its Laplace transform by X(s).

(a) Using eq. (9.3), evaluate X(s) and specify its region of convergence.

Leplace Transform

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

$$X(s) = \int_{-\infty}^{\infty} e^{-5t} * u(t-1)e^{-s*t}dt$$

$$X(s) = \int_{1}^{\infty} e^{-5t} * e^{-st}dt$$

$$X(s) = -\frac{e^{-(s+5)t}}{s+5}|_1^\infty$$

$$X(s) = -\frac{1}{s+5}[0 - e^{-(s+5)}]$$

$$X(s) = \frac{e^{-(s+5)}}{s+5}$$

The region of convergence is Re{s} $>$ -5

Thus, the Laplace transform of x(t) is $\frac{e^{-(s+5)}}{s+5}$ and the region of convergence is Re{s} $>$ -5

Q.3:(9.4) For the Laplace transform of

$$x(t) = \begin{cases} e^t sin(2t) & t \le 0 \\ 0 & t > 0 \end{cases}$$

indicate the location of its poles and its region of convergence.

$$x(t) = \begin{cases} e^t sin(2t) & t \le 0 \\ 0 & t > 0 \end{cases}$$

x(t) = [e^tsin(2t)] u(-t)

$[e^{-at}sin(\omega_0 t)]u(t)$ –Leplace transform$->$ $\frac{\omega_0}{(s+\alpha)^2+{\omega_0}^2}$

$$Re\{s\} > -\alpha$$

$[e^{-t}sin(2t)]u(t)$ –Leplace transform$->$ $\frac{2}{(s+1)^2+2^2}$

$$Re\{s\} > -1$$

x(at) Leplace transform–> $\frac{1}{|a|}X(\frac{s}{a})$

x(-t) Leplace transform–> X(-s)

$$[e^t sin(2t)]u(-t) \text{ – Leplace transform– } > \frac{-2}{(-s+1)^2+2^2}$$

$$[e^t sin(2t)]u(-t) \text{ – Leplace transform– } > \frac{-2}{(s-1)^2+2^2}$$

Find zero when $(s-1)^2+2^2$ = 0 to find poles of x(s)

$(s-1)^2+2^2$ = 0
$s^2-2*s+5=0$

$s=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. = $1\pm 2j$

The poles are $1\pm 2j$

Q.4:(9.5(a)) For each of the following algebraic expressions for the Laplace transform of a signal, determine the number of zeros located in the finite s-plane and the number of zeros located at infinity:

(a) $\frac{1}{s+1}$ + $\frac{1}{s+3}$

$$\frac{1}{s+1}+\frac{1}{s+3}=\frac{s+3+s+1}{s^2+3s+s+3}=\frac{2s+4}{s^2+4s+3}$$

It is zero when 2s+4 = 0, s = 2

poles when s^2+4s+3 = 0, s^2+4s+3 = (s+1)(s+3). then poles are s = -1 , -3

The order of denominator polynomial exceeds the order of numerator polynomial by 1. Therefore, one zero is in the finite s-plane and one zero located at infinity.

Q.5:(9.19(a)) Determine the unilateral Laplace transform of each of the following signals, and specify the corresponding regions of convergence:

$$x(t)=e^{-2t}*u(t+1)$$

$$X(s)=\int_{0^-}^{\infty}x(t)e^{-st}dt$$

$$X(s)=\int_{0^-}^{\infty}e^{-2t}*u(t+1)e^{-st}dt$$

$$X(s)=\int_{0^-}^{\infty}e^{-(s+2)t}dt$$

$$X(s)=\frac{1}{s+2} \qquad Re>-2$$